

# A simple Local Interstellar Spectrum model to fit the proton fluxes measured by the AMS and PAMELA detectors

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## Abstract

In this paper we discuss some simple analytical models to fit the cosmic-ray (CR) proton data collected by the AMS detector in June 1998 and by the PAMELA detector in several campaigns covering the period 2006-2009. The CR proton spectrum at Earth is derived starting from the model of the local interstellar spectrum (LIS) and folding it with the solar modulation potential in the force field approximation. The data are well described by a LIS modeled with a simple power law particle momentum density.

*Keywords:*

Cosmic Ray protons, Local Interstellar Spectrum

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## 1. Introduction

Cosmic rays (CRs) interact with gas atoms during their propagation in the interstellar medium and can suffer significant energy losses, thus modifying their injection spectra and composition. In addition, the spectra of CRs reaching the Earth are affected by the solar wind and the by the solar magnetic field (solar modulation effect). The solar modulation plays a relevant role on CR spectra in the low energy region, and its effect needs to be disentangled to allow a comprehensive picture to emerge. In fact, to understand the origin and propagation of CRs, a knowledge of their energy spectra in the interstellar medium is required.

Precise measurements of CR spectra over a wide rigidity range, from a few hundred MV to

tens of GV can be used to study the effect of solar modulation, including the convective and adiabatic cooling effect of the expanding solar wind and the diffusive and particle drift effects of the turbulent heliospheric magnetic field (HMF).

A full three-dimensional (3D) model was developed to compute the differential intensity of CR protons from 10 MeV to 30 GeV at Earth [1], and was applied to give an interpretation of the PAMELA proton data sets collected from 2006 to 2009 [2]. The model also includes a detailed treatment of the CR propagation in the solar magnetic field, that allows a precise description of the solar modulation effect. The implementation of this approach requires to provide the local interstellar proton spectrum (LIS) as initial condition. The LIS input spectrum is then “modulated” by the solar magnetic field, that affects the shape of the CR spectrum at Earth.

The choice of the LIS has always been rather contentious (see for instance [3]), and its parametrization in terms of proton kinetic energy could be complex (see for instance [1], [4], [5]).

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In this paper we assume some simple analytical LIS models to fit the proton fluxes measured by the AMS detector in June 1998 [6] and by the PAMELA detector in different periods from 2006 to 2009. The solar modulation effect is also described in a simple form, using the force-field approximation [7].

## 2. Local proton spectrum models

The simplest model describing CR acceleration is the first-order Fermi mechanism, where particles gain energy by diffusing back and forth across a shock front while convecting downstream. In this framework, the particle differential density per unit momentum  $n(p)$  is  $\propto p^{-a}$ . After injection into the interstellar medium with spectral index  $a \simeq 2.1 \div 2.2$ , characteristic of supernova remnant (SNR) shocks, CRs are transported in the astronomical environments with rigidity-dependent escape lengths, that soften their spectra by  $\delta \simeq 0.5 \div 0.6$ , leaving a steady-state CR particle momentum density  $n(p) \propto p^{-\alpha}$ , where  $\alpha = a + \delta \simeq 2.7 \div 2.8$  [8].

Therefore, in the present work we will consider a simple LIS model with the differential particle momentum density in the form:

$$n(p) = k_0 \left( \frac{p}{p_0} \right)^{-\alpha}. \quad (1)$$

The spectral differential intensity in momentum is obtained by multiplying  $n(p)$  for the factor  $\beta c/4\pi$ , where  $\beta c$  is the particle velocity:

$$J(p) = \frac{\beta c}{4\pi} n(p) = k\beta \left( \frac{p}{p_0} \right)^{-\alpha}. \quad (2)$$

where  $k = k_0 c/4\pi$ . Hereafter we will assume  $c = 1$  and we will express both energies and momenta in units of GeV. In writing the previous equations we introduced a momentum scale  $p_0 = 1$  GeV. Therefore  $k_0$  will be expressed in the same units as  $n(p)$ , i.e. in  $\text{GeV}^{-1}\text{m}^{-3}$  and  $k$  will be expressed in the same units as  $J(p)$ , i.e. in  $\text{GeV}^{-1}\text{m}^{-2}\text{s}^{-1}\text{sr}^{-1}$ .

The differential intensity in momentum can be converted into a differential intensity in kinetic energy taking into account that:

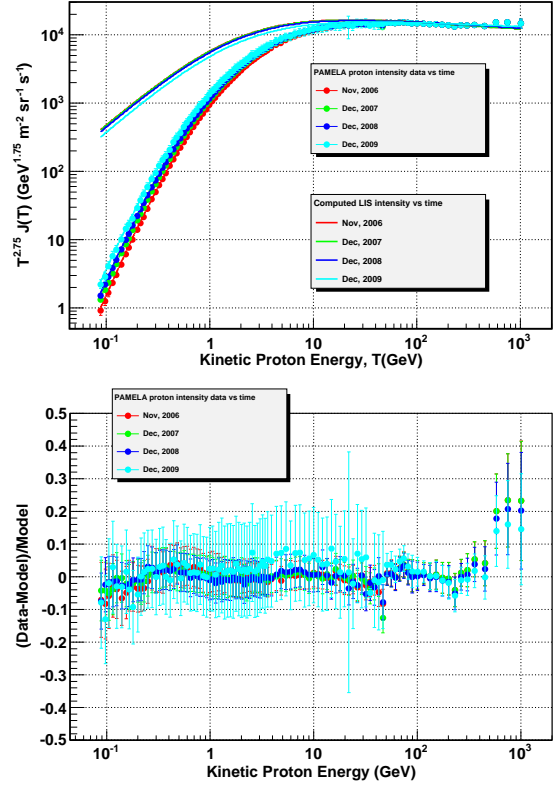


Figure 1: Fit of the PAMELA proton data collected in November 2006 (red), December 2007 (green), December 2008 (blue) and December 2009 (cyan) with a simple momentum power law LIS folded with solar modulation (the fit was performed up to 175 GeV). The top panel shows the fit results superimposed to the data points. The dashed lines show the fitted spectra (Eq. 7) and the continuous lines show the corresponding LIS (Eq. 5). The bottom panel shows the fit residuals.

$$J(T) = \frac{dp}{dT} J(p) \quad (3)$$

$$\frac{dp}{dT} = \frac{T + m}{\sqrt{T(T + 2m)}} = \frac{1}{\beta} \quad (4)$$

where  $m$  is the particle rest mass. The CR spectrum in the interstellar space as a function of the kinetic energy is therefore given by:

$$J_{LIS}(T) = k \left[ \frac{T(T + 2m)}{p_0^2} \right]^{-\frac{\alpha}{2}} \quad (5)$$

The spectral index in kinetic energy,  $\alpha_T$ , is defined as:

$$\alpha_T = -\frac{d \log J_{LIS}(T)}{d \log T} = \alpha \frac{T + m}{T + 2m}. \quad (6)$$

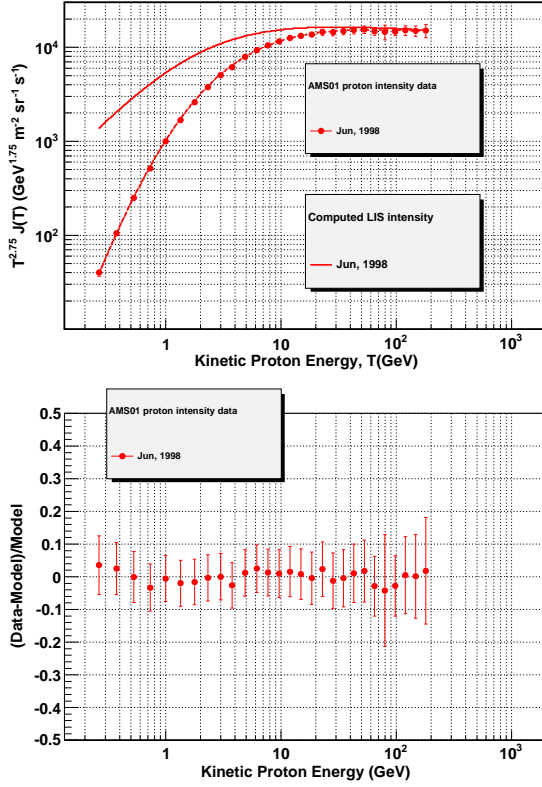


Figure 2: Fit of the AMS proton data collected in June 1998 (red) with a simple momentum power law LIS folded with solar modulation. The top panel shows the fit results superimposed to the data points. The dashed lines show the fitted spectra (Eq. 7) and the continuous lines show the corresponding LIS (Eq. 5). The bottom panel shows the fit residuals vs energy.

The previous result shows that the CR spectrum exhibits a change of curvature with increasing kinetic energy ( $\alpha_T \rightarrow \alpha/2$  for  $T \rightarrow 0$  while  $\alpha_T \rightarrow \alpha$  for  $T \rightarrow \infty$ ).

The LIS model of Eq. 5 can be folded with the solar modulation effect in the force field approximation introducing the solar modulation potential field  $\Phi$ , and results into a CR spectrum at Earth given by:

$$J_{Earth}(T) = J_{LIS}(T + \Phi Z/A) \frac{T(T + 2m)}{(T + m + \Phi Z/A)^2 - m^2} \quad (7)$$

where  $Z$  and  $A$  are the atomic and mass number respectively of the given CR species.

Figure 1 shows the results obtained fitting the PAMELA proton data with Eq. 7. The data up to 48 GeV have been taken from Table 1

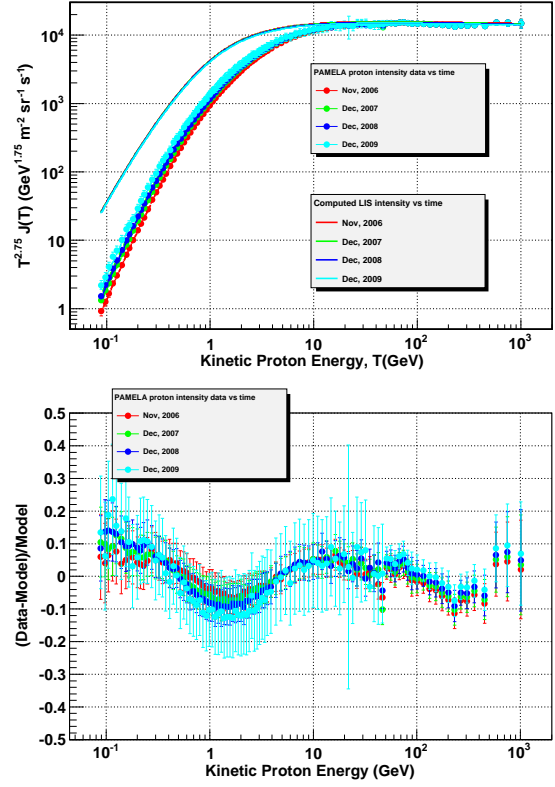


Figure 3: Fit of the PAMELA proton data collected in November 2006 (red), December 2007 (green), December 2008 (blue) and December 2009 (cyan) with the LIS of Eq. 8 folded with solar modulation (the fit was performed up to 175 GeV). The top panel shows the fit results superimposed to the data points. The dashed lines show the fitted spectra and the continuous lines show the corresponding LIS (Eq. 8). The bottom panel shows the fit residuals.

of Ref. [2]. The data at higher energies, from 48 GeV to 1 TeV have been taken from Ref. [9]. The fit was performed up to 175 GeV using the MINUIT package implemented in the ROOT framework [10]. The residuals exhibit very small fluctuations, within a few %. It is worth to point out that the 2009 data show some large point-to-point fluctuations, probably due to the fact that these data were collected at the end of the 11<sup>th</sup> Solar cycle. Figure 2 shows the results obtained fitting the AMS proton data with Eq. 7. The data have been taken from Table 3 of Ref. [6]. The error bars shown in the plots are evaluated by adding in quadrature statistical and systematic uncertainties. Also in this case the residuals exhibit small fluctuations.

The results of all the fits are summarized in Table 1. It is interesting to point out that both the AMS and PAMELA data sets are well fitted by the momentum power law LIS. Moreover, the values of the LIS parameters (prefactor and spectral index) obtained from the fit of the AMS data are consistent with those obtained from the fits of the PAMELA data sets taken in 2006, 2007 and 2008, as one would expect since the LIS is time independent. On the other hand, the fit of the data collected by PAMELA in 2009 yields values that differ significantly from those obtained from the other fits. Because of the consistency between the AMS and the first three PAMELA fits, we decided to combine these results into a unique LIS, with a spectral index  $\alpha = 2.853$  and a pre-factor of  $2.4863 \times 10^4 \text{ GeV}^{-1}\text{m}^{-2}\text{s}^{-1}\text{sr}^{-1}$ , evaluated from the weighted average of the individual fit results. Using these parameters for the LIS (Eq. 5) and leaving only the solar modulation potential free, the fits of the AMS and of the PAMELA data do not change significantly with respect to the ones shown in Figures 1 and 2. In particular, the fitted values of the solar modulation potential do not exhibit significant variations (see Table 1).

We have also fitted the data samples using the LIS model given in Ref. [1], i.e.:

$$J_{LIS}(T) = \begin{cases} 707 k e^{4.64-0.08(\log T)^2-2.91\sqrt{T}} & T < T_0 \\ 685 k e^{3.22-2.78\log T-1.5/T} & T > T_0 \end{cases} \quad (8)$$

where  $T_0 = 1.4 \text{ GeV}$ . The numerical coefficients in the previous equation include units of measurement:  $J_{LIS}(T)$  is given in units of  $\text{GeV}^{-1}\text{m}^{-2}\text{s}^{-1}\text{sr}^{-1}$  if  $T$  is expressed in  $\text{GeV}$ . The coefficient  $k$  is a scale factor ( $k = 1$  will reproduce exactly the formula in Eq. 13 of Ref. [1]).

Figure 3 shows the results of the fits performed assuming the LIS model in Eq. 8 folded with the solar modulation in the force field approximation. In this case the residuals show larger fluctuations with respect to the simple power law fits and the  $\chi^2$  values are higher. The fit results are summarized in Table 1. It is also worth to point out that, when this LIS model is assumed, the

solar modulation potential values are smaller with respect to those obtained assuming the simple momentum power law model. This feature is due to the shape of the LIS given in Eq. 8, that predicts a curvature at low energies. Another interesting feature of these fits is the value of the prefactor, that in all cases is consistent with 1, as expected. We have also performed the fit of the AMS and PAMELA data with the LIS model of Eq. 8 with a fixed prefactor  $k = 1$ . This constraint does not worsen the fit, and the solar modulation potential values do not change significantly.

To reproduce the B/C ratio the injection spectrum of CR protons is often approximated with a power law in the rigidity space with a break at a few GV (see for instance [3]). A possible description of this feature can be given by choosing for the LIS momentum density a broken power law function (with a discontinuity in the first derivative at the break):

$$n_{LIS}(p) = \begin{cases} k_0 \left(\frac{p}{p_b}\right)^{-\alpha_1} & p < p_b \\ k_0 \left(\frac{p}{p_b}\right)^{-\alpha_2} & p \geq p_b \end{cases} \quad (9)$$

that result into a differential intensity given by:

$$J_{LIS}(T) = \begin{cases} k \left(\frac{p}{p_b}\right)^{-\alpha_1} & p < p_b \\ k \left(\frac{p}{p_b}\right)^{-\alpha_2} & p \geq p_b \end{cases} \quad (10)$$

with  $p = \sqrt{T(T+2m)}$  and  $k = k_0 c/4\pi$ . The break momentum  $p_b$  corresponds to a break kinetic energy  $T_b = \sqrt{p_b^2 + m^2} - m$ .

The results of the fits with the LIS of Eq. 10 are also shown in Table 1. We note that the fits yield no evidence of a break, since  $\alpha_1 \approx \alpha_2$  within the errors for all the data sets.

### 3. Conclusions

The simple power law model of the proton LIS folded with the solar modulation in the force field

approximation provides a good fit of the both AMS and PAMELA proton data. This result needs to be investigated with further analyses, for instance by using data from helium and heavy nuclei on short time periods, that are not publicly available.

In general, it is worth to point out that a CR measurement at Earth does not allow to easily reconstruct the LIS, since the Solar modulation effect cannot be easily disentangled. However, a constraint to the LIS spectrum could be provided by a fit of the gamma-ray emissivity of the local neutral gas measured by the Fermi LAT [8, 11, 12].

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Simple power law LIS (Eq. 5)					
	AMS Jun 1998	PAMELA Nov 2006	PAMELA Dec 2007	PAMELA Dec 2008	PAMELA Dec 2009
$k(10^4 \text{ GeV}^{-1} \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1})$	$2.342 \pm 0.193$	$2.548 \pm 0.067$	$2.520 \pm 0.065$	$2.414 \pm 0.064$	$2.072 \pm 0.097$
$\alpha$	$2.832 \pm 0.024$	$2.858 \pm 0.007$	$2.857 \pm 0.007$	$2.847 \pm 0.007$	$2.818 \pm 0.011$
$\Phi(\text{GeV})$	$0.629 \pm 0.030$	$0.685 \pm 0.009$	$0.602 \pm 0.008$	$0.557 \pm 0.008$	$0.454 \pm 0.012$
$\chi^2/d.o.f.$	1.4/25	16.2/87	15.1/87	11.1/87	28.2/87

Simple power law LIS (Eq. 5) with $k = 2.4863 \times 10^4 \text{ GeV}^{-1} \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1}$ and $\alpha = 2.853$					
	AMS Jun 1998	PAMELA Nov 2006	PAMELA Dec 2007	PAMELA Dec 2008	PAMELA Dec 2009
$\Phi(\text{GeV})$	$0.646 \pm 0.010$	$0.676 \pm 0.003$	$0.598 \pm 0.003$	$0.566 \pm 0.003$	$0.501 \pm 0.005$
$\chi^2/d.o.f.$	2.4/27	17.8/89	15.5/89	12.5/89	59.3/89

LIS from ref. [1] (Eq. 8)					
	AMS Jun 1998	PAMELA Nov 2006	PAMELA Dec 2007	PAMELA Dec 2008	PAMELA Dec 2009
$k$	$1.062 \pm 0.025$	$1.044 \pm 0.078$	$1.029 \pm 0.080$	$1.015 \pm 0.080$	$0.995 \pm 0.089$
$\Phi(\text{GeV})$	$0.583 \pm 0.018$	$0.594 \pm 0.005$	$0.500 \pm 0.005$	$0.459 \pm 0.005$	$0.377 \pm 0.007$
$\chi^2/d.o.f.$	11.6/26	56.9/88	82.3/88	104.4/88	54.1/88

LIS from ref. [1] (Eq. 8) with $k = 1$					
	AMS Jun 1998	PAMELA Nov 2006	PAMELA Dec 2007	PAMELA Dec 2008	PAMELA Dec 2009
$\Phi(\text{GeV})$	$0.550 \pm 0.011$	$0.572 \pm 0.004$	$0.488 \pm 0.003$	$0.453 \pm 0.003$	$0.379 \pm 0.007$
$\chi^2/d.o.f.$	17.9/27	88.3/89	95.3/89	107.6/89	54.4/89

Broken power law LIS (Eq. 10)					
	AMS Jun 1998	PAMELA Nov 2006	PAMELA Dec 2007	PAMELA Dec 2008	PAMELA Dec 2009
$k(\text{GeV}^{-1} \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1})$	$134.38 \pm 25.18$	$157.24 \pm 16.30$	$155.69 \pm 10.14$	$153.36 \pm 10.42$	$132.67 \pm 29.38$
$\alpha_1$	$2.705 \pm 0.155$	$2.942 \pm 0.089$	$2.888 \pm 0.053$	$2.844 \pm 0.051$	$2.908 \pm 0.112$
$\alpha_2$	$2.824 \pm 0.024$	$2.863 \pm 0.012$	$2.858 \pm 0.007$	$2.847 \pm 0.007$	$2.817 \pm 0.012$
$\Phi(\text{GeV})$	$0.566 \pm 0.080$	$0.722 \pm 0.039$	$0.615 \pm 0.023$	$0.556 \pm 0.021$	$0.484 \pm 0.043$
$p_b(\text{GeV})$	$6.13 \pm 0.41$	$5.96 \pm 2.11$	$5.94 \pm 0.14$	$5.91 \pm 0.14$	$6.00 \pm 0.47$
$\chi^2/d.o.f.$	1.0/23	14.9/85	14.9/85	11.1/85	27.6/85

Table 1: Results of the fits of the AMS and PAMELA proton data with the various LIS models discussed in the text.